IGCP
Agência de Gestão da Tesouraria
e da Dívida Pública

The Portuguese Bond Index

A measure for bond portfolio performance

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PORTUGUESE TREASURY AND DEBT MANAGEMENT AGENCY

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Abstract

We propose a benchmark methodology that can be used to measure performance in Portuguese bond portfolios. The suggested Portuguese Bond Index (PBI) aims to reflect the behaviour of a diversified bond portfolio that can be seen as a proxy for the Portuguese bond market. This study also compares the proposed PBI with a fictional investors' investment portfolio that considers a certain established trading bond strategy. The comparison is performed both on an absolute and risk adjusted returns basis. Although all implementations are made only for PBI, the proposed methodologies for index construction and performance evaluation can be easily extended to other kind of bond indexes if, having into account the investor's preferences and or constraints, they are considered better benchmarks.

Key words: Benchmark, bond index, performance evaluation.

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1 Introduction

When investing in financial assets, reaching the higher possible return may be the major goal for a risk-taker investor. A more correct approach to measure performance advises that portfolio results should be analysed by considering the achieved absolute returns in association with some risk metrics, i.e., risk-adjusted returns[1]. Additionally, the investor must evaluate the performance of the returns or the risk-adjusted returns against an appropriate benchmark. This approach provides the investors with comprehension on how the market is performing, but also how they are performing, as investors.

Choosing the appropriate benchmark, as declared by (Brown, 2006) should respect the following very important attribute: the benchmark must be relevant to the investment strategy deployed by the investor. It is crucial that the benchmark matches the investor requirements. For example, if an investor owns German stocks a German index such as the DAX30 should be used as benchmark. In case of investment in American technological stocks a technological index like NASDAQ100 may be the best choice. For a stock portfolio comprehending securities from several countries, a global benchmark such as the MSCI World should be used. If we are a bond portfolio manager trading in Eurozone bonds then an index such as the Bloomberg Eurozone Government Index >1yr[2] (EUGATR Index) could be used as performance benchmark. This principle is transversal to all types of financial assets[3], stocks, bonds, commodities, real estate, etc.

Given this information, the following question arises: Which benchmark should be used to measure performance of an investor solely investing in Portuguese bonds?

[1] Risk-adjusted return refines an investment's return by measuring how much risk is involved in producing that return. Some common risk-adjusted measures are the Sharpe, Treynor, Information and Sortino ratios.

[2] This index consists of fixed-rate investment-grade public obligations of the sovereign countries participating in the European Monetary Union.

[3] As example, the Bloomberg commodity index (BCOM) could be used as benchmark to commodities, and the MSCI World IMI Core Real Estate Index for real estate. The S&P US Aggregate Bond Index can be used as benchmark for bonds investor in the United States.

An index that comprises the main Portuguese sovereign bonds available in the market could be the answer. It would reflect the most liquid bond market in Portugal and it is expected to measure the direction and amplitude of moves in the Portuguese market as a whole. Such index is already available and can be found in a Bloomberg terminal using the ticker "PTGATR Index", which stands for Portuguese government bonds > 1 year EFFAS[4] standardized rules total return index.

In this study we aim at two major objectives. Firstly, we present in detail the methodology and rational to create the benchmark for Portuguese bonds comparing afterwards its figures with those provided by Bloomberg. We believe that providing a clear understanding, both practical and theoretical, on how to build an actual bond index is advantageous as it allows the readers to reproduce it. By doing this study we are both providing a way to create a benchmark and validating Bloomberg index calculations. We took the initiative of calling this index the Portuguese Bond Index (PBI).

Secondly, we simulate several Portuguese bond purchases like an investor would do in the real world, and then we compare his performance to what would have been obtained by similar purchases of the benchmark Portuguese Bond Index. By using this methodology, or a similar approach, an investor can have a neutral measure of the difference between what he gets and what he could expect from investing in the market.

2 Benchmark rational and methodology

A benchmark is a standard against which the performance of a security, investment fund or portfolio manager can be measured. Additionally, the PBI can be interpreted as a market proxy.

[4] European Federation of Financial Analyst' Societies.

There are three main approaches to construct a benchmark: price weighted series[5], market equal weighted series[6] and unweighted series[7]. As stated by (Bacon, 2011) the market equal weighted indexes (also known as arithmetic calculated index), in which each security is represented in the index according with its market value weight, is the most common approach used in the financial markets, and therefore the one used by us in our study.

Next, we describe in detail the rational and methodology used to construct the PBI.

2.1 The benchmark rational

As pointed out by (Brown, 2002), in order to create the Portuguese Bond Index first we must generate a set of rules for inclusion, exclusion and rebalancing of securities in the benchmark. The rules for the index constituents' selection are as follows;

- Only Portuguese Republic government securities are allowed to be included in the Portuguese Bond Index.
- The only securities allowed to constitute the PBI are fixed rated coupon treasury bonds (OTs). Treasury bills (BTs), variable yield treasury bonds (OTRVs) and other types of securities are excluded.
- Euro currency denominated instruments only.
- Bonds are excluded from the index when they reach one year to maturity[8].
- The number of constituents in the index is subject to the selection and eligibility criteria at the time of rebalance.

[5] A price weighted series is the average of the bonds' prices in the series. It is calculated by adding together each bond price and dividing the total by the number of bond in the series.
[6] A market weighted series is the market value (bond price multiplied by bond outstanding) of each bond in the series divided by the sum of all index bond market value.
[7] An unweighted series is based on the average price movement of the bond prices in the index. In this series, all bonds, no matter what the price, have the same effect on the series.
[8] This is due to the facts that these securities are, by that time, regarded as money market instruments and often harder to trade.

- All inflows and outflows are taken into consideration in the index construction, such as new bond issuances, coupon payments or other types of money flows. Any flows of cash are reinvested in the index, using a total return index methodology.
- The index is market-capitalization weighted with daily calculations[9] and daily rebalancing[10]. The calculations are made considering 4 pm bond prices[11].

2.2 The benchmark methodology

As referred in the previous rules section, an index must allow for the constituents to change freely. There must be the ability to add new issuances, remove issued securities in case they are redeemed or called, and to permit the change of the outstanding amount in a bond due to further tranche issuances or partial calls. Allowing these changes to occur prevents the index value from presenting a gap or jump in the time series when calculations are performed.

A chain-link index calculation method adequately solves this problem. Using this methodology the index value is defined by the previous calculation times the aggregate percentage change in the value of the current constituents since the previous calculation. This method must allow for the constituents to change their issuance size, and this is achieved by weighting both the current and previous prices by the outstanding amount at

[9] PBI is calculated on a daily basis, although ignoring weekends. This convention allows us to not underestimate the index volatility.

[10] This is the most significant difference between PBI and the Bloomberg PTGATR Index. Although PTGATR Index is built like PBI, considering the EFFAS standardized rules, Bloomberg rebalances the Index constituents once per month at the end of that period while PBI rebalances daily. For instance, if a new bond is issued at 15 of March, Bloomberg will only consider this new bond as a PTGATR Index constituent at 1 of April. A similar rational is applied by Bloomberg if a bond is removed from the index as a consequence of maturity range below 1 year being achieved. For example, if a bond reaches this limit at 12 of July, PTGATR Index removes the bond from the index constituents at the beginning of the month, 1 of July. PBI methodology will include or exclude bonds from the index at the precise day the event occurs. Bloomberg methodology might exclude bonds suffering from liquidity shortages. As no precise methodology for this exclusion type is presented by Bloomberg or any other literature we do not consider it.

[11] PTGATR Index uses 00:38 am bond prices in index calculations. This information was provided by Bloomberg helpdesk directly. This hour aims to gather end of day quotes as well as eventually OTC transactions.

the previous date. Considering all this information, for a certain date t the clean price index value can be calculated by the following equation;

$$CI_t = CI_{t-1} * \frac{\sum_i P_{i,t} * N_{i,t-1}}{\sum_i P_{i,t-1} * N_{i,t-1}}, \tag{1}$$

$$CI_0 = 100$$

Where CI_{t-1} symbolizes the clean price index value at the previous day, $P_{i,t}$ and $P_{i,t-1}$ represents the clean price of the i[th] bond at time t and t-1 respectively. $N_{i,t-1}$ denotes the nominal value of outstanding amount at time t-1 for the i[th] bond. We declare that the index at inception date, t_0 starts with value 100.

A clean price index shows the capital performance, ignoring any income received. In this type of index it is assumed that the bonds are quoted without any accrued interest.

It is straightforward to understand that a clean price index cannot be used for performance measurement because it does not accurately indicate if a fund manager has good or bad performance against the index benchmark. This is due to the fact that proceeds from coupon payments are ignored when calculating the clean price index, although redemptions and other types of cash flows are still accounted for.

A second type of index calculation that quickly comes across as being complementary for the clean price methodology is the gross price index method. For any bond, the gross price is the clean price plus the accrued interest, and by using this methodology the accrued interests are added in index calculations. The gross price index is given by;

$$GI_t = CI_t * (1 + AI_t), \tag{2}$$

Where CI_t represents the clean price index for t using equation (1) and AI_t being obtained as follows:

$$AI_t = \frac{\sum_i A_{i,t} * N_{i,t-1}}{\sum_i P_{i,t} * N_{i,t-1}}, \qquad (3)$$

Where NI_{t-1} represents the nominal value of outstanding amount at time t-1 for the i[th] bond, while $P_{i,t}$ characterizes the clean price of the i[th] bond at time t. $A_{i,t}$ denotes the accrued interest of the i[th] bond for time t. We point out that, in contrast to the clean price index methodology, the gross price index will most likely not present at t_0 the value of 100. This is due to the fact that gross index takes into account the existence of accrued interest in the bond index constituents at inception date.

We believe this methodology introduces an improvement as it considers the existence of accrued interest and an investor when purchasing a security must always pay for the gross price. However, it still does not take into consideration the coupons received by the investor. This type of index will have discontinuities in time that are not justified by any price movements. When the i[th] bond pays interest, the effect of that payment will cause a drop to zero in the accrued interest $A_{i,t}$.

We can overcome this problem by using a total return index methodology. This is the methodology used in our study. It assumes that all coupons received are re-invested in the index. This is the process generally used when dealing with bond index creation, and it means that we must reinvest the proceeds in the index in proportion to the size of the holdings[12]. It also has the significant advantage that index performance will not be dependent

[12] In a total return index the proceeds are usually reinvested proportionally to the size of the holdings. Alternative methods for investing proceeds are: (1) Remove the proceeds from calculations – This option should only be considered if we only want to measure capital performance. It is used in clean price indexes type. (2) Proceeds go to cash balance not earning any interest – Not a very satisfactory solution as this will create over time a considerable cash balance not earning any interest. (3) Proceeds in cash that earn interest – This improves the solution in the second method but means that an external rate of interest should be chosen and specified. External factors are not desirable. (4) Coupon payments are reinvested in the security that has produced the cash-flow – This approach means that over time the securities with higher coupons will get disproportionately large weightings compared to zero or low coupon bonds. It also has the disadvantage that its performance is dependent on the base date of the index.

on the base date of the index[13]. The total returns index will be the methodology used in our study to create the PBI benchmark and it is calculated as follows:

$$PBI_t = TI_t = TI_{t-1} * \frac{\sum_i \left(P_{i,t} + A_{i,t} + G_{i,t}\right) * N_{i,t-1}}{\sum_i \left(P_{i,t-1} + A_{i,t-1}\right) * N_{i,t-1}}, \qquad (4)$$

$$PBI_0 = TI_0 = 100$$

Where TI_{t-1} represents the total return index value at the previous day, and $G_{i,t}$ stands for the value of any coupon payment received from the i[th] bond at time t since time t-1. This approach will keep the index continuous through time as well as it will account for all income received by the investor.

Next, we present several numerical examples to help understanding the PBI construction previously stated.

2.3 Total return benchmark practical creation

Consider that the moment of the index inception, t_0 is 01-01-2014. This was arbitrarily chosen because it is irrelevant for the analysis and implementation of the methodology, despite its effective results will depend on the start date. Please see appendix 6, section 6.1 for further information on this subject.

Although the index will present at t_0 the value of 100, the first step should be gathering all the relevant information on the index constituents in order to satisfy the rules referred at section 2.1. This enable us to construct the index for t_1 using equation (4). The relevant information is presented in Table 1.

[13] The daily change for an index starting in 2014 (100 at this point) or an index starting in 2017 (at 500, for instance) is the same for any given date in 2017, even though the 2 indexes may have different values.

Table 1 – Benchmark constituents at t_0= 01/01/2014.

1	2	3	4	5	6	7	8
Valuation Date t = t_0	**Instrument at valuation date t_0**	**Nominal Amount t_0**	**Price**	**Accrued Interest**	**Cash flow**	**$(P_{i,0} + A_{i,0}) \times N_{i,0}$**	**$(P_{i,0} + A_{i,0} + G_{i,0}) \times N_{i,0}$**
01-01-2014	OT_3.35_15OCT2015	9.241.692.650	100,85	0,72	0	9.386.176.387	9.386.176.387
01-01-2014	OT_3.6_15OCT2014	0	101,20	0,00	0	0	0
01-01-2014	OT_3.85_15APR2021	7.510.407.000	88,45	2,75	0	6.849.717.525	6.849.717.525
01-01-2014	OT_4.1_15APR2037	6.972.589.000	72,90	2,93	0	5.287.612.723	5.287.612.723
01-01-2014	OT_4.2_15OCT2016	6.185.000.000	101,13	0,90	0	6.310.402.993	6.310.402.993
01-01-2014	OT_4.35_16OCT2017	11.258.380.100	99,66	0,92	0	11.322.853.604	11.322.853.604
01-01-2014	OT_4.375_16JUN2014	0	101,36	0,00	0	0	0
01-01-2014	OT_4.45_15JUN2018	10.853.007.000	98,55	2,44	0	10.960.273.364	10.960.273.364
01-01-2014	OT_4.75_14JUN2019	7.664.750.000	96,86	2,62	0	7.624.759.429	7.624.759.429
01-01-2014	OT_4.8_15JUN2020	8.550.891.000	95,39	2,63	0	8.381.808.844	8.381.808.844
01-01-2014	OT_4.95_25OCT2023	7.227.758.000	92,30	0,92	0	6.737.874.424	6.737.874.424
01-01-2014	OT_5.65_15FEB2024	3.000.000.000	96,25	3,59	0	2.995.236.986	2.995.236.986
01-01-2014	OT_6.4_15FEB2016	3.500.000.000	106,13	5,61	0	3.910.846.062	3.910.846.062
					∑	79.767.562.341	79.767.562.341

As can be observed, only treasury bonds are selected. The values presented for OT_3.6_15OCT2014 and OT_4.375_16JUN2014 are zero because their remaining maturity in less than one year (at t_0). Therefore, at t_0, the Portuguese Bond Index is composed of eleven treasury bonds since the two referred bonds are excluded. We then display for each of these instruments the nominal value of outstanding amount, the price (clean market price) and the accrued interest. The cash-flow column only shows values different from zero if the bond pays a coupon on this date. If that is the case, the coupon will be represented in percentage (of the nominal value). Column 7 represents the sum of nominal amount multiplied by the clean price plus the accrued interest. Column 8 exhibits the sum of nominal amount multiplied by the clean price plus the accrued interest and cash flows representing the coupons received on this date.

The information shown in Table 1 can be straightforward used in equation (4) in order to construct the index for t_1. In column 7 the number presented as the sum of all instruments for t_0, 79.767.562.341, represents directly the denominator of the equation: $\sum_i \left(P_{i,t-1} + A_{i,t-1}\right) * N_{i,t-1}$. However the numerator information cannot be obtained from the Table 1, as it only discloses data for t_0 . It is then necessary to gather information for t_1, 02-01-2014, as it follows:

Table 2 - Benchmark constituents at t_1= 02/01/2014.

1	2	3	4	5	6	7	8
Valuation Date t = t₁	**Instrument at valuation date t₁**	**Nominal Amount t_0**	**Price**	**Accrued Interest**	**Cash flow**	**$(P_{i,1} + A_{i,1})$ x $N_{i,0}$**	**$(P_{i,1} + A_{i,1} + G_{i,1})$ x $N_{i,0}$**
02-01-2014	OT_3.35_15OCT2015	9.241.692.650	101,06	0,73	0	9.406.894.236	9.406.894.236
02-01-2014	OT_3.6_15OCT2014	0	101,47	0,00	0	0	0
02-01-2014	OT_3.85_15APR2021	7.510.407.000	89,65	2,76	0	6.940.259.082	6.940.259.082
02-01-2014	OT_4.1_15APR2037	6.972.589.000	73,78	2,94	0	5.349.580.414	5.349.580.414
02-01-2014	OT_4.2_15OCT2016	6.185.000.000	101,63	0,91	0	6.341.885.067	6.341.885.067
02-01-2014	OT_4.35_16OCT2017	11.258.380.100	100,32	0,93	0	11.398.500.665	11.398.500.665
02-01-2014	OT_4.375_16JUN2014	0	101,45	0,00	0	0	0
02-01-2014	OT_4.45_15JUN2018	10.853.007.000	99,42	2,45	0	11.055.475.049	11.055.475.049
02-01-2014	OT_4.75_14JUN2019	7.664.750.000	97,95	2,63	0	7.708.727.816	7.708.727.816
02-01-2014	OT_4.8_15JUN2020	8.550.891.000	96,52	2,64	0	8.479.130.869	8.479.130.869
02-01-2014	OT_4.95_25OCT2023	7.227.758.000	93,75	0,94	0	6.843.476.424	6.843.476.424
02-01-2014	OT_5.65_15FEB2024	3.000.000.000	97,68	3,61	0	3.038.601.370	3.038.601.370
02-01-2014	OT_6.4_15FEB2016	3.500.000.000	106,35	5,63	0	3.919.247.260	3.919.247.260
					∑	80.481.778.251	80.481.778.251

Prices $P_{i,t}$, accrued interest $A_{i,t}$ and cash-flows $G_{i,t}$ are now updated to t_1. We underline the fact that the nominal amount in Table 1 is the same as the one displayed in Table 2, as equation (4) requires the notional to be referred to t-1, $N_{i,t-1}$, both in numerator and denominator. Therefore, the numerator of the equation $\sum_i \left(P_{i,t} + A_{i,t} + G_{i,t}\right) * N_{i,t-1}$, can be directly obtained in column 8 by using the number 80.481.778.251. As the total return index at t_0 takes the following value;

$$PBI_0 = TI_0 = 100$$

Then at date t_1 the PBI value is as follows:

$$PBI_1 = TI_1 = 100 * \frac{80.481.778.251}{79.767.562.341} = 100{,}895$$

Let's now perform the index calculations a bit further in time, specifically at t_{74}, which is 15-04-2014. On this date there are two bonds in the index paying coupons: OT_3.85_15APR2021 and OT_4.1_15APR2037. Consequently, there are two cash-flows that must be dealt with by the index in order to make PBI a total return index type. Using the chosen approach, the proceeds are reinvested in the index constituents according to their weights.

The first step should be gathering all the relevant information for date t_{74} as shown in Table 3. This is necessary to calculate equation (4) numerator. We also display in Table 4 the relevant information for t_{73}, used in the denominator calculations. As can be observed in Table 3 there are two rows with grey background. These rows exhibit the instruments that are paying coupon at date t_{74}, and consequently the accrued interest goes to zero on this date. However, this proceeds are booked as cash-flow $G_{i,t}$ in equation (4).

Table 3 - Benchmark constituents at t_{74}= 15/04/2014.

1	2	3	4	5	6	7	8
Valuation Date t = t_{74}	**Instrument at valuation date t_{74}**	**Nominal Amount t_{73}**	**Price**	**Accrued Interest**	**Cash flow**	**($P_{i,74}$ + $A_{i,74}$) x $N_{i,73}$**	**($P_{i,74}$ + $A_{i,74}$ + $G_{i,74}$) x $N_{i,73}$**
15-04-2014	OT_3.35_15OCT2015	7.918.013.650	103,64	1,67	0	8.338.411.693	8.338.411.693
15-04-2014	OT_3.6_15OCT2014	0	101,45	0,00	0	0	0
15-04-2014	OT_3.85_15APR2021	7.510.407.000	104,30	0,00	3,85	7.833.542.261	8.122.692.931
15-04-2014	OT_4.1_15APR2037	6.972.589.000	94,45	0,00	4,10	6.585.610.311	6.871.486.460
15-04-2014	OT_4.2_15OCT2016	6.185.000.000	106,72	2,09	0	6.730.315.776	6.730.315.776
15-04-2014	OT_4.35_16OCT2017	11.258.380.100	108,11	2,16	0	12.414.010.406	12.414.010.406
15-04-2014	OT_4.375_16JUN2014	0	100,59	0,00	0	0	0
15-04-2014	OT_4.45_15JUN2018	10.853.007.000	108,46	3,71	0	12.173.416.540	12.173.416.540
15-04-2014	OT_4.75_14JUN2019	10.914.750.000	110,23	3,97	0	12.464.554.790	12.464.554.790
15-04-2014	OT_4.8_15JUN2020	8.550.891.000	110,46	4,00	0	9.786.734.877	9.786.734.877
15-04-2014	OT_4.95_25OCT2023	7.227.758.000	109,39	2,33	0	8.075.039.357	8.075.039.357
15-04-2014	OT_5.65_15FEB2024	6.000.000.000	114,40	0,91	0	6.918.647.260	6.918.647.260
15-04-2014	OT_6.4_15FEB2016	3.500.000.000	109,45	1,03	0	3.867.045.719	3.867.045.719
					∑	95.187.328.989	95.762.355.808

Table 4 - Benchmark constituents at t_{73}= 14/04/2014.

1	2	3	4	5	6	7	8
Valuation Date t = t_{73}	**Instrument at valuation date t_{73}**	**Nominal Amount t_{73}**	**Price**	**Accrued Interest**	**Cash flow**	**$(P_{i,73} + A_{i,73}) \times N_{i,73}$**	**$(P_{i,73} + A_{i,73} + G_{i,73}) \times N_{i,73}$**
14-04-2014	OT_3.35_15OCT2015	7.918.013.650	103,61	1,66	0	8.335.704.825	8.335.704.825
14-04-2014	OT_3.6_15OCT2014	0	101,48	0,00	0	0	0
14-04-2014	OT_3.85_15APR2021	7.510.407.000	104,15	3,84	0	8.110.259.606	8.110.259.606
14-04-2014	OT_4.1_15APR2037	6.972.589.000	94,37	4,09	0	6.864.950.851	6.864.950.851
14-04-2014	OT_4.2_15OCT2016	6.185.000.000	106,59	2,08	0	6.721.563.577	6.721.563.577
14-04-2014	OT_4.35_16OCT2017	11.258.380.100	107,94	2,15	0	12.393.529.407	12.393.529.407
14-04-2014	OT_4.375_16JUN2014	0	100,65	0,00	0	0	0
14-04-2014	OT_4.45_15JUN2018	10.853.007.000	108,26	3,69	0	12.150.116.026	12.150.116.026
14-04-2014	OT_4.75_14JUN2019	10.914.750.000	110,13	3,96	0	12.452.219.627	12.452.219.627
14-04-2014	OT_4.8_15JUN2020	8.550.891.000	110,20	3,98	0	9.764.019.377	9.764.019.377
14-04-2014	OT_4.95_25OCT2023	7.227.758.000	108,87	2,32	0	8.036.655.507	8.036.655.507
14-04-2014	OT_5.65_15FEB2024	6.000.000.000	114,09	0,90	0	6.899.418.493	6.899.418.493
14-04-2014	OT_6.4_15FEB2016	3.500.000.000	109,43	1,02	0	3.865.644.521	3.865.644.521
					∑	95.594.081.817	95.594.081.817

The total return index at t_{73}, 14/04/2014, has the following value[14]:

$$PBI_{73} = TI_{73} = 113{,}253$$

Then at date t_{74} the PBI value is:

$$PBI_{74} = TI_{74} = 113{,}253 * \frac{95.762.355.808}{95.594.081.817} = 113{,}452$$

Next we exhibit PBI calculations considering the case where a bond reaches the limit of less than one year to maturity. This situation occurs for the first time in our index at t_{205}, which corresponds to 15-10-2014. On this date, two bonds pay coupons: OT_4.2_15OCT2016 and OT_3.35_15OCT2015 as presented in Table 5. Moreover, the latest security reaches a maturity of less than one year and therefore has to be excluded from index from t_{206} onwards. As can be observed in Table 6, which refers to date t_{206}, the instrument OT_3.35_15OCT2015 no longer exhibits nominal amount.

[14] Please see appendix 6.2 to check total return index value evolution from inception until t_{74}

Table 5 - Benchmark constituents at t_{205}= 15/10/2014.

1	2	3	4	5	6	7	8
Valuation Date t = t_{205}	**Instrument at valuation date t_{205}**	**Nominal Amount t_{204}**	**Price**	**Accrued Interest**	**Cash flow**	**($P_{i,205}$ + $A_{i,205}$) x $N_{i,204}$**	**($P_{i,205}$ + $A_{i,205}$ + $G_{i,205}$) x $N_{i,204}$**
15-10-2014	OT_3.35_15OCT2015	6.618.669.650	103,10	0,00	3,35	6.823.682.942	7.045.408.376
15-10-2014	OT_3.6_15OCT2014	0	0,00	0,00	0	0	0
15-10-2014	OT_3.85_15APR2021	7.510.407.000	108,13	1,93	0	8.265.599.000	8.265.599.000
15-10-2014	OT_3.875_15FEB2030	3.500.000.000	99,38	0,37	0	3.491.305.137	3.491.305.137
15-10-2014	OT_4.1_15APR2037	6.972.589.000	99,02	2,06	0	7.047.412.999	7.047.412.999
15-10-2014	OT_4.2_15OCT2016	6.185.000.000	106,82	0,00	4,20	6.606.817.000	6.866.587.000
15-10-2014	OT_4.35_16OCT2017	11.258.380.100	109,20	4,34	0	12.782.267.392	12.782.267.392
15-10-2014	OT_4.45_15JUN2018	10.853.007.000	110,07	1,49	0	12.107.060.809	12.107.060.809
15-10-2014	OT_4.75_14JUN2019	10.914.750.000	112,37	1,60	0	12.439.342.465	12.439.342.465
15-10-2014	OT_4.8_15JUN2020	9.692.519.000	113,76	1,60	0	11.181.714.796	11.181.714.796
15-10-2014	OT_4.95_25OCT2023	7.227.758.000	113,58	4,81	0	8.557.078.835	8.557.078.835
15-10-2014	OT_5.65_15FEB2024	7.725.000.000	118,90	3,75	0	9.474.405.616	9.474.405.616
15-10-2014	OT_6.4_15FEB2016	3.500.000.000	107,78	4,24	0	3.920.727.568	3.920.727.568
					∑	102.697.414.560	103.178.909.994

Table 6 - Benchmark constituents at t_{206}= 16/10/2014.

1	2	3	4	5	6	7	8
Valuation Date t = t_{206}	**Instrument at valuation date t_{206}**	**Nominal Amount t_{205}**	**Price**	**Accrued Interest**	**Cash flow**	**($P_{i,206}$ + $A_{i,206}$) x $N_{i,205}$**	**($P_{i,206}$ + $A_{i,206}$ + $G_{i,206}$) x $N_{i,205}$**
16-10-2014	OT_3.35_15OCT2015	0	102,86	0,00	0	0	0
16-10-2014	OT_3.85_15APR2021	7.510.407.000	106,90	1,94	0	8.174.013.188	8.174.013.188
16-10-2014	OT_3.875_15FEB2030	3.500.000.000	97,23	0,38	0	3.416.339.212	3.416.339.212
16-10-2014	OT_4.1_15APR2037	6.972.589.000	95,86	2,07	0	6.828.211.038	6.828.211.038
16-10-2014	OT_4.2_15OCT2016	6.185.000.000	106,42	0,01	0	6.582.788.699	6.582.788.699
16-10-2014	OT_4.35_16OCT2017	11.258.380.100	108,61	0,00	4	12.228.008.086	12.717.747.620
16-10-2014	OT_4.45_15JUN2018	10.853.007.000	109,34	1,50	0	12.029.699.683	12.029.699.683
16-10-2014	OT_4.75_14JUN2019	10.914.750.000	111,60	1,61	0	12.356.992.171	12.356.992.171
16-10-2014	OT_4.8_15JUN2020	9.692.519.000	112,68	1,62	0	11.078.067.910	11.078.067.910
16-10-2014	OT_4.95_25OCT2023	7.227.758.000	111,66	4,83	0	8.419.105.391	8.419.105.391
16-10-2014	OT_5.65_15FEB2024	7.725.000.000	116,88	3,76	0	9.319.363.279	9.319.363.279
16-10-2014	OT_6.4_15FEB2016	3.500.000.000	107,70	4,26	0	3.918.541.267	3.918.541.267
					∑	94.351.129.925	94.840.869.459

Once again, we have to provide data for t-1 in order to be able to calculate the index for t_{205}. Information regarding t_{204} is provided in Table 7.

Table 7 - Benchmark constituents at t_{204}= 14/10/2014.

1	2	3	4	5	6	7	8
Valuation Date t = t_{204}	**Instrument at valuation date t_{204}**	**Nominal Amount t_{204}**	**Price**	**Accrued Interest**	**Cash flow**	**$(P_{i,204} + A_{i,204})$ x $N_{i,204}$**	**$(P_{i,204} + A_{i,204} + G_{i,204})$ x $N_{i,204}$**
14-10-2014	OT_3.35_15OCT2015	6.618.669.650	103,13	3,34	0	7.046.621.043	7.046.621.043
14-10-2014	OT_3.6_15OCT2014	0	0,00	0,00	0	0	0
14-10-2014	OT_3.85_15APR2021	7.510.407.000	109,52	1,92	0	8.369.764.745	8.369.764.745
14-10-2014	OT_3.875_15FEB2030	3.500.000.000	101,39	0,36	0	3.561.283.562	3.561.283.562
14-10-2014	OT_4.1_15APR2037	6.972.589.000	100,99	2,04	0	7.184.338.409	7.184.338.409
14-10-2014	OT_4.2_15OCT2016	6.185.000.000	107,20	4,19	0	6.889.223.676	6.889.223.676
14-10-2014	OT_4.35_16OCT2017	11.258.380.100	109,87	4,33	0	12.856.638.246	12.856.638.246
14-10-2014	OT_4.45_15JUN2018	10.853.007.000	110,97	1,48	0	12.203.143.372	12.203.143.372
14-10-2014	OT_4.75_14JUN2019	10.914.750.000	113,73	1,59	0	12.586.908.390	12.586.908.390
14-10-2014	OT_4.8_15JUN2020	9.692.519.000	115,31	1,59	0	11.330.431.895	11.330.431.895
14-10-2014	OT_4.95_25OCT2023	7.227.758.000	115,40	4,80	0	8.687.463.134	8.687.463.134
14-10-2014	OT_5.65_15FEB2024	7.725.000.000	120,81	3,73	0	9.620.757.329	9.620.757.329
14-10-2014	OT_6.4_15FEB2016	3.500.000.000	107,89	4,23	0	3.924.051.370	3.924.051.370
					∑	104.260.625.170	104.260.625.170

The total return index at t_{204} has the following value:

$$PBI_{204} = TI_{204} = 119{,}327$$

By using the sum values presented in Table 5 column 8 and in Table 7 column 7 the PBI value for t_{205} can be obtained as follows:

$$PBI_{205} = TI_{205} = 119.327 * \frac{103.178.909.994}{104.260.625.170} = 118{,}089$$

When a new issuance, buy back or exchange offer of given securities are performed, the PBI calculations should follow the same methodology as the one we have just explained above, and for this reason we will not describe them thoroughly apart from emphasizing that the nominal amount considered in the index calculations for each date t should always be referred to t-1, as stated in equation (1).

2.3.1 Gross and Clean price index benchmark

As an alternative to the total return index, the clean price index or the gross price index could be used as benchmark. This task can be properly executed by using the information provided in the following tables.

Table 8 – Clean and dirty price indexes constituents at t_{74}= 15/04/2014.

1	2	3	4	5	6	7
Valuation Date t = t_{74}	**Instrument at valuation date t_{74}**	**Nominal Amount t_{73}**	**Price**	**Accrued Interest**	**$P_{i,74}$ x $N_{i,73}$**	**$A_{i,74}$ x $N_{i,73}$**
15-04-2014	OT_3.35_15OCT2015	7.918.013.650	103,64	1,67	8.206.031.396	132.380.297
15-04-2014	OT_3.6_15OCT2014	0	101,45	0,00	0	0
15-04-2014	OT_3.85_15APR2021	7.510.407.000	104,30	0,00	7.833.542.261	0
15-04-2014	OT_4.1_15APR2037	6.972.589.000	94,45	0,00	6.585.610.311	0
15-04-2014	OT_4.2_15OCT2016	6.185.000.000	106,72	2,09	6.600.786.625	129.529.151
15-04-2014	OT_4.35_16OCT2017	11.258.380.100	108,11	2,16	12.171.153.267	242.857.139
15-04-2014	OT_4.375_16JUN2014	0	100,59	0,00	0	0
15-04-2014	OT_4.45_15JUN2018	10.853.007.000	108,46	3,71	11.771.171.393	402.245.147
15-04-2014	OT_4.75_14JUN2019	10.914.750.000	110,23	3,97	12.031.328.925	433.225.865
15-04-2014	OT_4.8_15JUN2020	8.550.891.000	110,46	4,00	9.444.886.654	341.848.223
15-04-2014	OT_4.95_25OCT2023	7.227.758.000	109,39	2,33	7.906.444.476	168.594.881
15-04-2014	OT_5.65_15FEB2024	6.000.000.000	114,40	0,91	6.863.850.000	54.797.260
15-04-2014	OT_6.4_15FEB2016	3.500.000.000	109,45	1,03	3.830.837.500	36.208.219
				∑	93.245.642.808	1.941.686.182

Table 9 - Clean and dirty price indexes constituents at t_{73}= 14/04/2014.

1	2	3	4	5	6
Valuation Date t = t_{73}	**Instrument at valuation date t_{73}**	**Nominal Amount t_{73}**	**Price**	**Accrued Interest**	**$P_{i,73}$ x $N_{i,73}$**
14-04-2014	OT_3.35_15OCT2015	7.918.013.650	103,61	1,66	8.204.051.893
14-04-2014	OT_3.6_15OCT2014	0	101,48	0,00	0
14-04-2014	OT_3.85_15APR2021	7.510.407.000	104,15	3,84	7.821.901.130
14-04-2014	OT_4.1_15APR2037	6.972.589.000	94,37	4,09	6.579.857.925
14-04-2014	OT_4.2_15OCT2016	6.185.000.000	106,59	2,08	6.592.746.125
14-04-2014	OT_4.35_16OCT2017	11.258.380.100	107,94	2,15	12.152.014.020
14-04-2014	OT_4.375_16JUN2014	0	100,65	0,00	0
14-04-2014	OT_4.45_15JUN2018	10.853.007.000	108,26	3,69	11.749.194.053
14-04-2014	OT_4.75_14JUN2019	10.914.750.000	110,13	3,96	12.020.414.175
14-04-2014	OT_4.8_15JUN2020	8.550.891.000	110,20	3,98	9.423.295.654
14-04-2014	OT_4.95_25OCT2023	7.227.758.000	108,87	2,32	7.869.040.829
14-04-2014	OT_5.65_15FEB2024	6.000.000.000	114,09	0,90	6.845.550.000
14-04-2014	OT_6.4_15FEB2016	3.500.000.000	109,43	1,02	3.830.050.000
				∑	93.088.115.805

Considering that we are again at date t_{74}, the clean price index can be directly computed by using the data presented both in Table 8 column 6 and Table 9 column 6 respectively. We emphasize that the nominal amount considered in the calculations should always be referred to t-1, as stated in equation (1).

Supposing that the clean price index at t_{73} is[15]:

$$CI_{73} = 112{,}237$$

Then at date t_{74} the clean price index value is directly obtained through equation (1):

$$CI_{74} = 112{,}237 * \frac{93.245.642.808}{93.088.115.805} = 112{,}427$$

The methodology to be applied when calculating the gross price index was previously exhibited in equations (2) and (3). Using information in Table 6 columns 6 and 7, and assuming that the clean price index at t_{74}, as previously calculated, is[16]:

$$CI_{74} = 112.427$$

At t_{74} the accrued index value, using equation (3) can be obtained as follows:

$$AI_{74} = \frac{1.941.686.182}{93.245.642.808} = 0{,}0208$$

Therefore at date t_{74} the gross price index value is achieved by equation (1):

$$GI_{73} = 112{,}427 * (1 + 0{,}0208) = 114{,}768$$

In order to complement the previous index type calculations we display in appendix 6.2 the historical values for the three index methodologies from t_0 until t_{73}. We present these values to underpin the importance of including proceeds. The total returns index

[15] See appendix 6.2 to check clean price index value evolution from inception until t_{74}

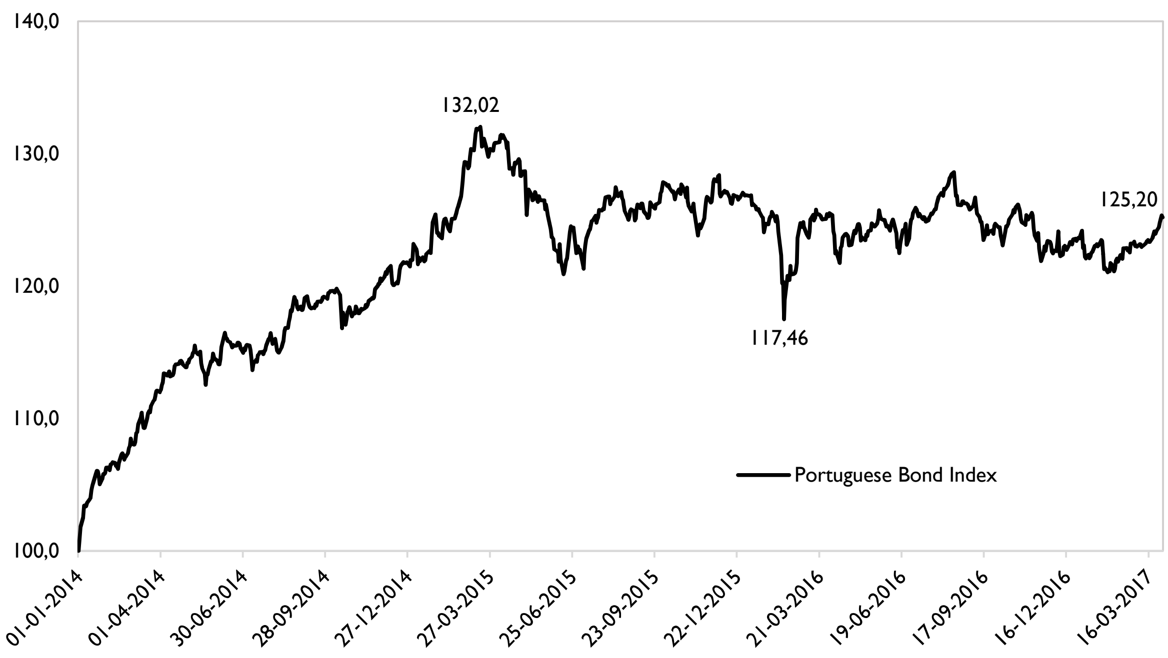
[16] See appendix 6.2 to check gross price index value evolution from inception until t_{74}

methodology in just 74 days already shows a significant divergence when compared to the clean or gross price index methodologies.

2.4 Benchmark results

The PBI evolution since inception is graphically displayed in Figure 1. It tells us how well the Portuguese bond market has performed and how it has been behaving during this period. A very strong bull market occurred in the first year and half, from January 2014 until mid-2015. In April 2015 the index topped, reaching a 32.02% positive valuation. In the following years PBI has been in a sideways performance, mainly as consequence of the bond prices decline during this period. The drop is justified by a rise in yields across European and US bond markets, albeit this trend had the beginning inflexion point early in Portugal, and also with a more pronounced bias (see appendix 6.3 for further details). Even though this event has a negative impact in bond prices, the coupons cancelled this effect and stabilized the index performance (See appendix 6.4). By the end of March 2017 PBI presented a 25.20% absolute return since inception.

Figure 1 – Portuguese Bond Index evolution since inception at t_0



In the next table we go deeper in PBI performance, presenting some further metrics such as drawdowns[17], Sharpe ratio[18], Value-at-risk[19], Skewness and Kurtosis[20], among others.

Table 10 – Portuguese Bond index Jan 2014-Mar 2017 performance metrics

	PBI		PBI
Absolute Returns	25,20%	Annualized St. Deviation	6,21%
Annualized Returns	7,17%	Skewness	-0,66
Sharpe Ratio	1,10	Kurtosis	3,94
Max. 1Y Drawdown	-11,01%	Min Daily Return	-2,36%
Historical VaR 95%	-0,610%	Max Daily Return	1,63%
Gaussian VaR 95%	-0,615%		

The annualized returns for de 2014-2017 investment period is fairly satisfactory, 7.17%, especially if it is taken into consideration that the index is, on its rawest definition, no more than a diversified portfolio of bonds (See appendix 6.5 for PBI constituents' changes over time).

Sharpe ratio[21], standard deviation and minimum and maximum daily returns present also good performance. The more severe 1 year drawdown experienced by PBI was -11.01%. The Skewness and Kurtosis values imply that negative returns are more common than positive returns in the index, and these returns produces more extreme outliers than the normal distribution. Value-at-risk (VaR) values are rather low, suggesting that PBI allows for a comfortable risk control.

[17]A drawdown is the peak-to-trough decline during a specific period of an investment. In our work the time period is 1 year. The drawdown is quoted as the percentage between the peak and the subsequent trough.

[18] It was developed by Nobel laureate William F. Sharpe. The Sharpe ratio is the average return earned in excess of the risk-free rate per unit of volatility or total risk. Value presented is the annualized Sharpe ratio.

[19] VaR represents the maximum potential loss which can occur to a portfolio of an investor, for certain level of confidence and for a given time period. All VaR results in our work consider a 1 day time frame. If the confidence level is 95%, this means that 5% of the times the loss will be larger than what VaR predicted. VaR does not give any information about the severity of loss. Gaussian VaR assumes that returns are normally distributed. Historical VaR considers the distribution of returns equal to the one presented by the historical returns.

[20] Skewness is a measure of the asymmetry of the probability distribution of a real-valued random variable about its mean. Kurtosis is a measure of the "tailedness" of the probability distribution of a real-valued random variable.

[21] Annualized daily Sharpe ratio considering data from Jan 2014 to Mar 2017

Overall, bearing in mind the results achieved, we consider that PBI accomplishes a good performance in the investment horizon considered.

Furthermore, we present in appendix 6.6 the performance comparison between the Portuguese Bond index computed in this study and the Blomberg index PTGATR provided by the Bloomberg platform. In order to make this comparison as accurate as possible we calculate PBI in appendix 6.6 using end of the day Bloomberg prices and not 4 pm prices as stated in 2.1. Using this approach we target to minimize possible divergences between prices among both methodologies. As expected, results are very similar as both of them try to replicate the same: the Portuguese government bonds performance.

Finally, as a complementary exercise, we graphically display in appendix 6.7 the PBI performance, from January 2014 to March 2017 considering the three index calculation methodologies stated at section 2.3: total return, clean price and dirty price indexes. It enlightens the importance of the coupon reinvestment in the benchmark performance.

3 Benchmark as performance measure

In this section we simulate the purchases of a fictional investor that is constructing a portfolio of Portuguese bonds and then compare the performance of his/her portfolio to the one obtained by the benchmark Portuguese Bond Index.

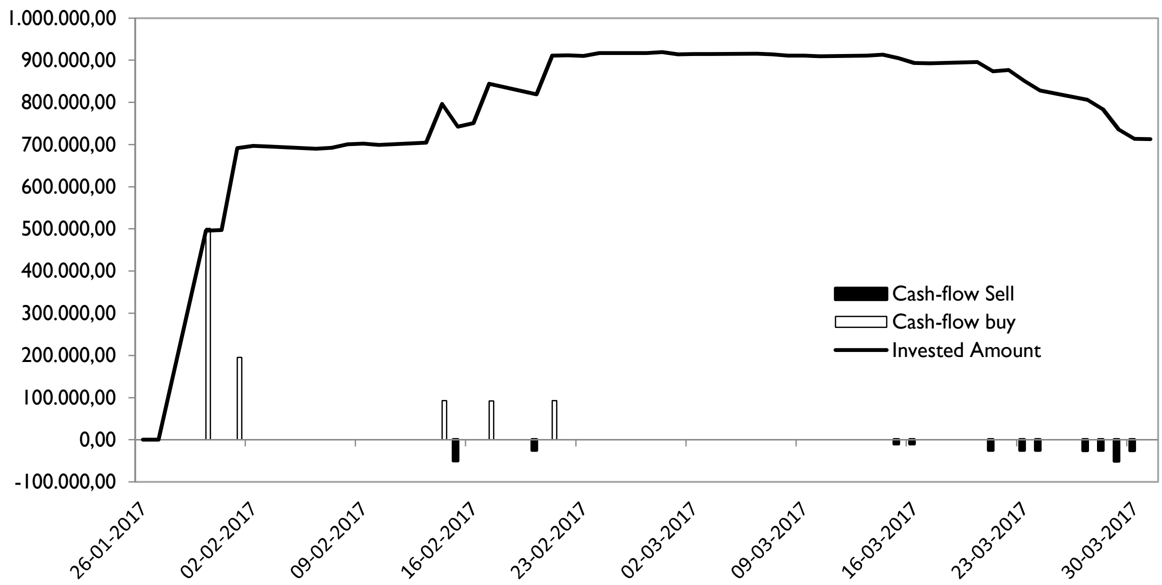
3.1 Creation of investors' portfolio

Our fictional trader will trigger some bond trades by following some predefined strategy[22]. Then, in order to successfully track his/her performance we should simulate purchases of equivalent amounts in our benchmark PBI. For instance, if the investor buys a €10.000

[22] The rational for the hypothetical investment strategy is not being under scrutiny in this study, only how to measure its performance. Possible transaction costs and commissions are not taken into account in our study.

amount of a Portuguese bond we should replicate this trade by purchasing the same amount in PBI. In the case of a sell transaction, the same percentage of the index should be sold. The Portuguese bond index plays as a tradable tracker that hypothetical can be bought or sold. In the following graph we present, chronologically, the trades performed by our fictional investor[23]:

Figure 2 – Fictional investor chronological trades



The fictional investor starts by triggering the highest trade in value in the creation of his/her portfolio, a transaction of around €500.000. The second trade is also the second most value trade performed during the time span under analysis, almost €200.000. All other trades, both purchases and sells, are lower in nominal amount, between €10.000 and €100.000.

The investor is directional on the buy side on the 10 years point of the Portuguese curve and is not constructing a diversified portfolio as he/she is using only one tradable instrument, the bond OT_4.125_14APR2027. All the fictional investor transactions amount,

23 Purchases are represented with positive values and sells with negative sells. The time span of our performance analysis is between Jan – Mar 2017.

cumulative positions market value and profit/losses over time are summarized[24] in the following table:

Table 11 – Investor's portfolio cash-flow, market value and profit & losses

Valuation Date t	Cash-flow	Cumulative Market value	Cumulative Profit & Losses
29-01-2017	0,00	0,00	0,00
30-01-2017	501.042,64	495.595,63	-5.447,01
01-02-2017	195.066,44	691.465,54	-4.643,54
14-02-2017	92.978,36	795.975,01	6.887,57
15-02-2017	-49.543,22	742.556,18	3.011,96
17-02-2017	92.381,99	844.007,36	12.081,16
20-02-2017	-24.761,99	819.219,22	12.055,00
21-02-2017	93.153,49	910.903,64	10.585,92
15-03-2017	-9.942,29	904.730,76	14.355,34
16-03-2017	-9.941,42	893.307,71	12.873,71
21-03-2017	-24.842,67	873.550,81	17.959,48
23-03-2017	-24.995,82	851.167,81	20.572,29
24-03-2017	-25.121,15	828.093,40	22.619,03
27-03-2017	-25.229,62	806.255,20	26.010,46
28-03-2017	-25.169,95	783.690,01	28.615,22
29-03-2017	-50.590,55	735.649,59	31.165,35
30-03-2017	-25.335,60	713.824,03	34.675,38
31-03-2017	0,00	713.056,50	33.907,85

As can be observed in the previous table, the overall performance of our strategy in terms of profit and loss went rather well. In a few months we obtained a near 5.0% profit. However, this is the moment when a benchmark is needed. Comparing our results to those obtained by a benchmark is of high importance as it tells us how well our strategy actually performed. Consequently, the next step should be to replicate the investor's purchases and sells of the bond in the benchmark index, allowing for the same investment amount in both strategies, otherwise a correct comparison cannot be fairly executed. Using the Portuguese bond index as benchmark tracker we obtained the results displayed in the following table:

[24] There are gaps between the dates displayed, as we only exhibit the values for the days when the investor actually intervened in the market by buying of selling a certain amount.

Table 12 – Index tracker price, trading units, market value and profit & losses

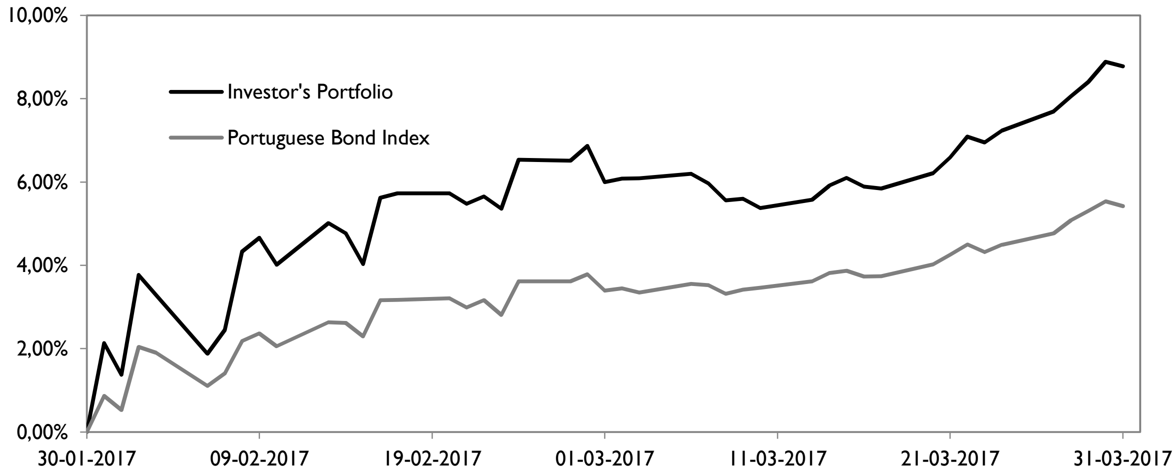
Valuation Date t	Total Return Index	Index trade Units	Cumulative Market value index	Cumulative Profit & Losses
30-01-2017	121,03	4139,83	501.042,64	0,00
01-02-2017	121,11	1610,72	696.422,03	312,95
14-02-2017	122,33	760,07	796.435,58	7.348,14
15-02-2017	122,06	-436,94	741.371,08	5.617,21
17-02-2017	122,85	752,01	838.512,56	10.376,70
20-02-2017	122,88	-200,47	814.098,37	10.595,71
21-02-2017	122,68	759,30	905.955,53	9.299,37
15-03-2017	123,47	-70,00	903.120,02	15.106,35
16-03-2017	123,33	-92,35	890.729,56	14.105,97
21-03-2017	123,87	-178,19	872.511,97	17.960,40
23-03-2017	123,93	-208,51	847.125,11	18.414,34
24-03-2017	124,12	-185,30	825.427,44	19.716,80
27-03-2017	124,44	-175,38	805.695,73	21.808,13
28-03-2017	124,80	-181,21	785.426,21	24.153,79
29-03-2017	125,06	-385,80	738.834,33	25.810,30
30-03-2017	125,35	-175,27	718.563,79	27.510,25
31-03-2017	125,20	0,00	717.721,96	26.668,42

The index units to be traded on each date are calculated in order to entail the same disbursements or reimbursements of those previously done by the investor in the bond. For example, as on 30/01/2017 the trader invested €-501.042 in the bond, we must buy the equivalent amount in the tracker. This means buying around 4140 trading units at the price of 121.03 which is the clean price of the PBI on that date.

Similarly to the fictional investor's portfolio, the PBI tracker also presents a fairly good performance in the same period. An absolute return[25] of 5.42% is achieved. Although this value is below the 8.78% obtained by the investor, it was attained with lower exposure to risk in terms of volatility. The following figure shows how absolute returns progressed during the investment period:

[25] In this study we are in the presence of inflows and outflows of cash and therefore we should measure returns using a money-weighted methodology. This approach internalizes in the returns both the timing and size of external cash flows (such as deposits and withdrawals). We use the Modified Dietz method. Note that the Modified Dietz return is a holding-period return, not an annual rate of return.

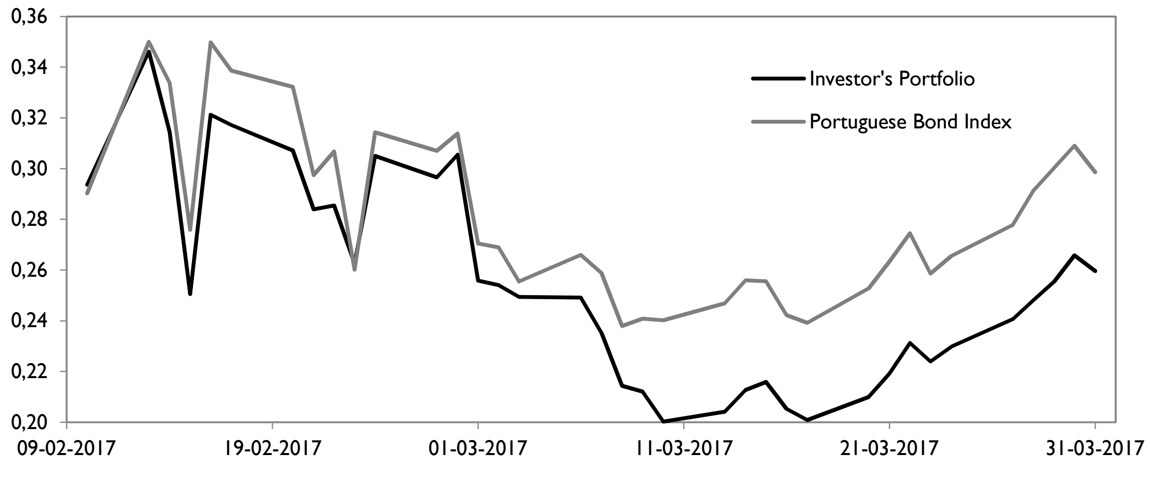
Figure 3 - Absolute returns dynamic performance



As can be observed, the absolute returns time series is more volatile for the investor's portfolio than for the benchmark. This can be a warning that gains (8.78% vs 5.42%) might have been obtained with higher exposure to risk in the investors' strategy (being volatility the metric for risk used in this case). This situation highlights the need of a risk-adjusted returns analysis.

The Sharpe ratio can be used to compare both strategies in terms of risk-adjusted returns as it stands for the average return earned in excess of the risk-free rate[26] per unit of volatility. In Figure 4 we exhibit the Sharpe ratio evolution for both strategies:

Figure 4 – Sharpe ratio risk adjusted returns dynamic performance

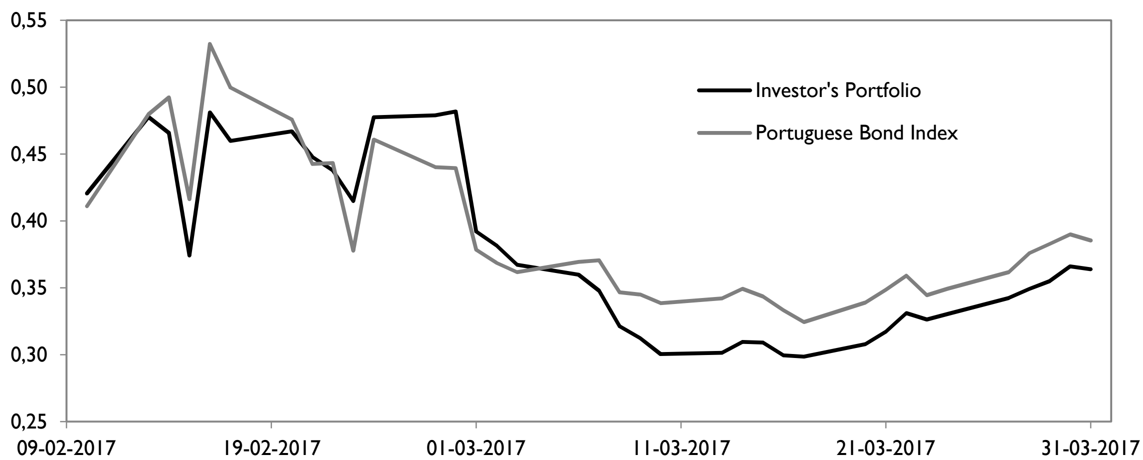


[26] A risk free rate of 0% is considered in this study.

As can be observed the Sharpe Ratio is more stable in PBI performance than the one obtained by the investor's portfolio and over time we can notice that PBI reaches higher risk-adjusted returns than the strategy employed by the investor. By the end of the time-series the PBI Sharpe ratio presents a value near 15% higher than the value obtained by trader's portfolio.

While the Sharpe ratio is the most widely used risk adjusted return metric, it is not without its limitations. Because of the way the Sharpe ratio is calculated, it tends to punish upside volatility. We believe the Sortino ratio improves on the Sharpe ratio in this subject as it only penalizes returns with downside volatility. The Sortino ratio takes the asset's excess return and then divides that amount by the asset's downside deviation. We define downside deviation as the asset's standard deviation of negative asset returns (downside volatility). In the following Figure we display the Sortino ratio progression for both strategies:

Figure 5 - Sortino ratio risk adjusted returns dynamic performance



In line with the results presented by the Sharpe ratio, over time we can notice that PIB Sortino ratio also reaches higher risk-adjusted returns than the investor's portfolio. However spreads in lower amount as the ratio is only is only 6% higher. This is a direct consequence of eliminating the effect of positive volatility and considering only negative volatility.

We can extend our analysis by using other risk metrics such as: Standard deviation, skewness, kurtosis and others as displayed in the following Table 13. These will help us to clarify which strategy deploys better performance over time.

In annualized terms, the returns are 61.5% and 35.15%, for the investor's strategy and the index respectively. A higher return for the investor was expected since he/she already had achieved higher absolute returns. However the index performs better in standard deviation, and minimum daily returns. This goes in line with the higher Sharpe and Sotino ratio presented by the index. Apparently we can infer that PBI performs better in terms of reducing portfolio risk.

Table 13 – Performance indicators

	Portuguese Bond Index	Investor's Portfolio
Absolute Returns	5,42%	8,78%
Annualized Returns	35,15%	61,50%
Sharpe Ratio	0,30	0,26
Sortino Ratio	0.39	0.36
Annualized St. Deviation	6,12%	11,03%
Skewness	1.14	1.16
Kurtosis	3.19	2.43
Min Daily Return	-0,78%	-1.37%
Max Daily Return	1.51%	2.37%

4 Conclusion

We have proposed a benchmark methodology that can be used to measure performance in Portuguese bond portfolios. The proposed Portuguese Bond Index aims to reflect the behaviour of a diversified bond portfolio that can be seen as a proxy for the Portuguese bond market. It is a market weighted and total return index. The alternative clean and gross price methods were illustrated. For the investment horizon considered in this study, the results show that the Portuguese Bond Index is expected to have overall good performance.

The results of PBI by the explained methodology were compared with those of Bloomberg for PTGATR index, and we conclude they are the same (allowing for only some implementation differences).

Although it was not developed here, the methodology for the construction of the index can be easily extended to other kind of indexes: equally weighted, fixed weights, all bonds or a selection of bonds with benchmark maturities, etc.

This study also compared PBI to a fictional investors' investment portfolio that used a certain established trading bond strategy. The comparison was performed both on an absolute and risk adjusted returns basis.

The PBI was used as the benchmark for our fictional investor but if another kind of index is considered to be a more relevant benchmark, the performance should be evaluated against it. The performance evaluation methodology implemented in this study can be easily applied in those cases, too.

The examples presented in this study are purposely uncomplicated as it was intended to be mainly expositive. For its effective implementation in real life; it is important to develop further research in performance measures and attribution methods, specifically considering investments in multiple maturity buckets. This will necessarily bring new challenges such as higher capability of database management and calculation power.

5 Bibliography

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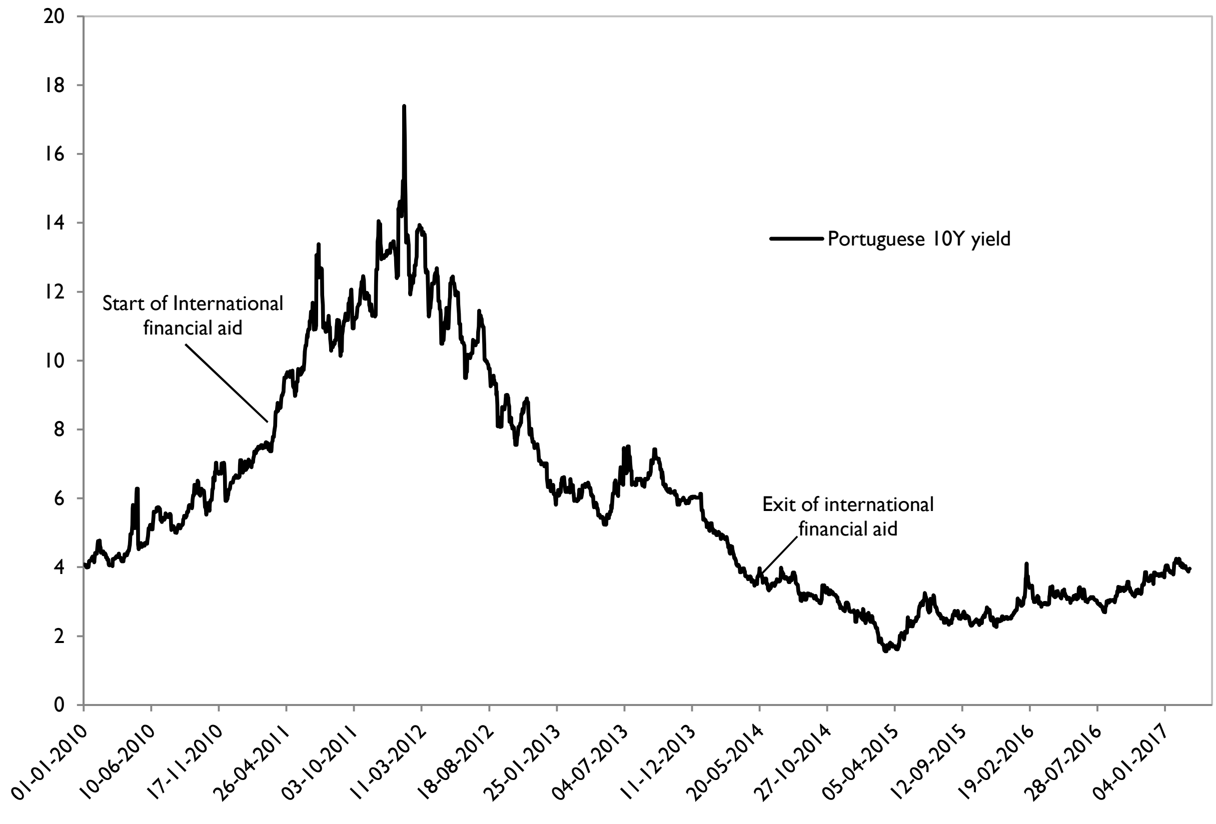
6 Appendix

6.1 Benchmark inception date selection process

In the following Figure 6 we display the Portuguese 10Y yield evolution since 2010. The period when the international financial assistance took place, translated into very high yields as a consequence of market perception of higher default risk. We arbitrarily chose 01-01-2014 as temporal starting point for the Portuguese Bond Index.

If a different starting date had been chosen, the index performance, period on period, would have been the same, despite the overall performance from start date until present would be very different. Ignoring coupons, if the yield on the starting date is higher (/lower) than on ending date, the performance (between starting and ending date) is positive (/negative). The coupon effect is explained in section 6.4.

Figure 6 – Portuguese 10y bond yield evolution from 2010 to 2017



6.2 Benchmark's evolution from t_0 to t_{74}

In the next table we present the evolution of the total return, clean price and gross price indexes since inception until t_{74}.

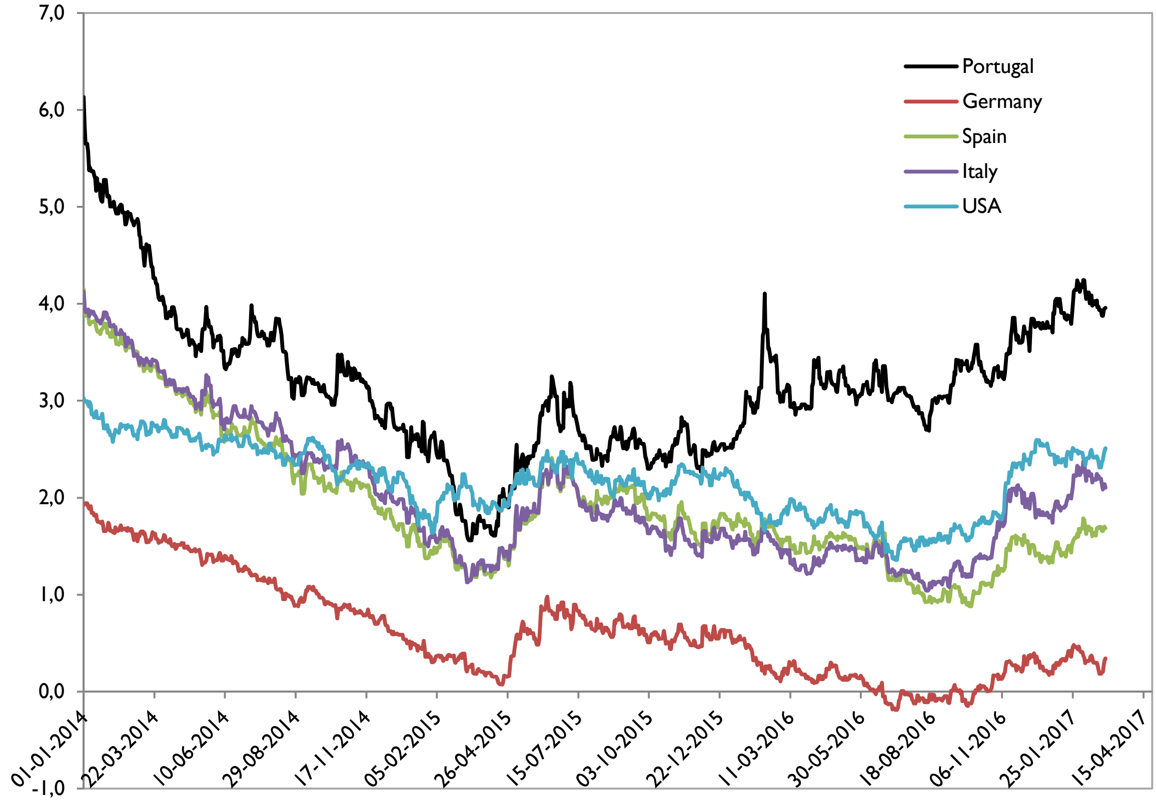
Table 14 – Indexes value evolution from t_0 to t_{74}

Number t	Valuation Date	Total Return Index	Clean Price Index	Dirty Price Index	Number t	Valuation Date	Total Return Index	Clean Price Index	Dirty Price Index
0	01-01-2014	100,000	100,000	102,174	38	24-02-2014	107,353	106,870	109,240
1	02-01-2014	100,895	100,902	103,088	39	25-02-2014	107,731	107,242	109,625
2	03-01-2014	101,819	101,833	104,032	40	26-02-2014	107,887	107,388	109,784
3	06-01-2014	102,501	102,491	104,729	41	27-02-2014	108,471	107,953	110,378
4	07-01-2014	103,397	103,394	105,645	42	28-02-2014	108,091	107,552	110,002
5	08-01-2014	103,430	103,416	105,679	43	03-03-2014	108,022	107,443	109,932
6	09-01-2014	103,354	103,404	105,601	44	04-03-2014	108,246	107,673	110,160
7	10-01-2014	103,591	103,634	105,834	45	05-03-2014	108,884	108,310	110,810
8	13-01-2014	103,857	103,869	106,106	46	06-03-2014	108,961	108,376	110,888
9	14-01-2014	103,992	103,994	106,243	47	07-03-2014	109,528	108,939	111,465
10	15-01-2014	104,580	104,583	106,844	48	10-03-2014	110,044	109,426	111,990
11	16-01-2014	104,923	104,809	107,195	49	11-03-2014	110,443	109,819	112,396
12	17-01-2014	105,171	105,049	107,448	50	12-03-2014	109,717	109,068	111,658
13	20-01-2014	105,827	105,680	108,117	51	13-03-2014	109,303	108,633	111,236
14	21-01-2014	106,042	105,888	108,338	52	14-03-2014	109,278	108,594	111,210
15	22-01-2014	106,018	105,850	108,313	53	17-03-2014	109,989	109,280	111,934
16	23-01-2014	105,649	105,460	107,936	54	18-03-2014	110,397	109,681	112,349
17	24-01-2014	105,016	104,802	107,290	55	19-03-2014	110,554	109,825	112,510
18	27-01-2014	105,429	105,184	107,711	56	20-03-2014	110,459	109,714	112,415
19	28-01-2014	105,786	105,537	108,076	57	21-03-2014	110,950	110,201	112,915
20	29-01-2014	105,794	105,532	108,084	58	24-03-2014	111,331	110,554	113,304
21	30-01-2014	105,892	105,620	108,185	59	25-03-2014	111,411	110,622	113,384
22	31-01-2014	106,275	105,997	108,575	60	26-03-2014	111,739	110,943	113,718
23	03-02-2014	106,309	105,994	108,611	61	27-03-2014	112,082	111,280	114,068
24	04-02-2014	106,073	105,740	108,369	62	28-03-2014	112,110	111,295	114,096
25	05-02-2014	106,518	106,182	108,824	63	31-03-2014	111,990	111,134	113,974
26	06-02-2014	106,549	106,201	108,856	64	01-04-2014	112,159	111,293	114,146
27	07-02-2014	106,696	106,338	109,005	65	02-04-2014	112,516	111,643	114,509
28	10-02-2014	106,640	106,243	108,949	66	03-04-2014	112,738	111,857	114,736
29	11-02-2014	106,403	106,065	108,706	67	04-04-2014	113,394	112,512	115,403
30	12-02-2014	106,337	105,986	108,622	68	07-04-2014	113,335	112,412	115,343
31	13-02-2014	106,190	105,823	108,471	69	08-04-2014	113,224	112,286	115,230
32	14-02-2014	106,669	106,300	108,960	70	09-04-2014	113,362	112,414	115,371
33	17-02-2014	107,274	106,881	109,160	71	10-04-2014	113,559	112,601	115,571
34	18-02-2014	107,373	106,969	109,261	72	11-04-2014	113,162	112,184	115,167
35	19-02-2014	107,254	106,834	109,139	73	14-04-2014	113,253	112,238	115,259
36	20-02-2014	106,892	106,453	108,771	74	15-04-2014	113,452	112,428	114,769
37	21-02-2014	107,045	106,596	108,927					

6.3 Several countries 10Y yields evolution 2014-2017

The next figure shows the evolution of 10Y bond yields for 4 European countries[27] (Portugal, Spain, Italy and Germany) and the US. It is shown that yields have risen in the last years for all countries, even though the inflection point of this new upwards trend differs among them. Portugal presents this point around Dec 2015 while the other countries only felt this new yields move in late 2016.

Figure 7 – Several countries 10y bond yields evolution from 2014 to 2017

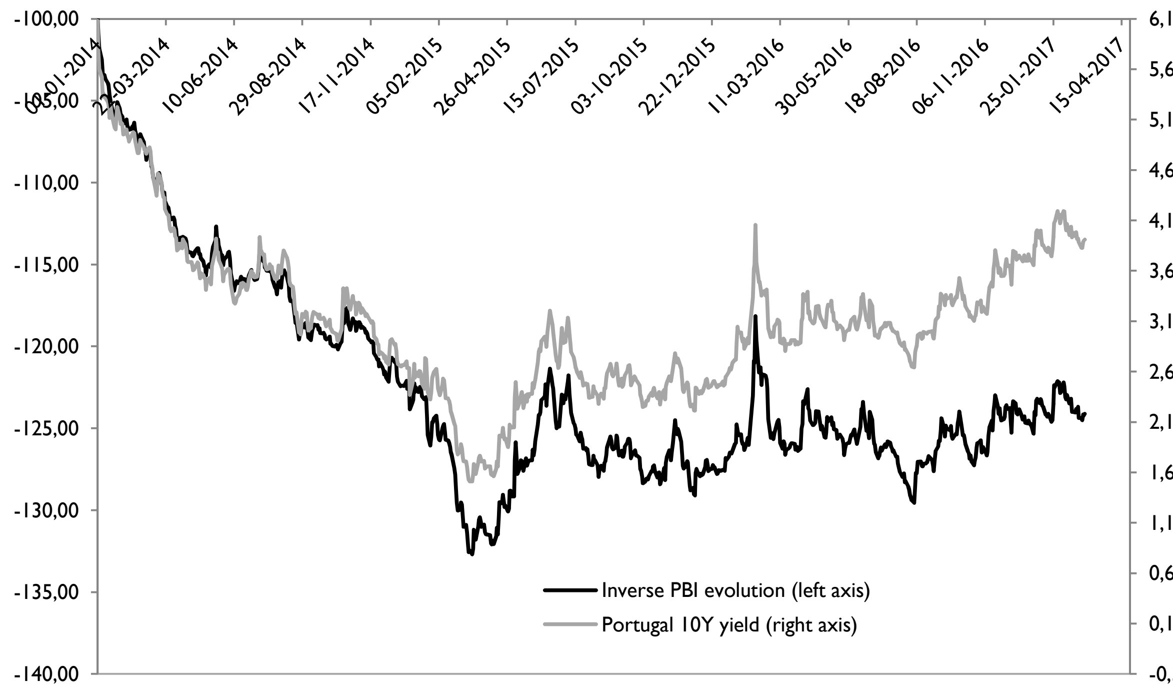


[27] Spain and Italy are commonly used as Portugal benchmarks while Germany is the main reference in Europe.

6.4 Inverse PBI vs 10y bond yield evolution

Figure 8 presented below tell us how PBI and Portuguese 10y bond yields behaved since index inception at 01-01-2014. For a better understanding we present PBI with negative values as this allow us to directly compare them to those achieved by the yield[28]. There is a high correlation between them (+0.88) in the data span analysed and it decays through time: at the beginning, correlation is near +1 but it presents a bigger divergence near the end. This is the "coupon effect", as bond yields do not take into account coupons after they are paid to those who own them while the index increases in value in order to hold these earnings. It is straight forward to conclude that even when market conditions are worst for bonds market value (when rates rise) the coupons can mitigate or even overcome this negative effect in the index performance.

Figure 8 – Performance of inverse PBI vs 10y bond yield from Jan 2014 to Feb 2017



[28] A consequence of the fact that bond prices increase when yields decrease and vice versa.

6.5 PBI constituents fluctuation over time

In the following table we show how index constituents changed over time every 6 months since inception. We display the weight of each of the bonds in the PBI. During this period, the percentage of individual bonds in the index range from a minimum of 2.5% to a maximum of 14%.

Besides the weight recompositions, we can see the entries in the index of newly issued bond lines and the exit of those that have entered in the range of less than 1 year to maturity.

Table 15 – PBI constituents weights from Jan 2014 to Jan 2017

Instrument	jan-2014	jul-2014	jan-15	jul-2015	jan-16	jul-2016	jan-17
OT_3.35_15OCT2015	11,77%	8,38%	-	-	-	-	-
OT_4.2_15OCT2016	7,91%	7,01%	5,60%	4,73%	-	-	-
OT_6.4_15FEB2016	4,90%	4,02%	3,25%	-	-	-	-
OT_4.35_16OCT2017	14,19%	12,98%	12,42%	9,25%	8,58%	7,85%	-
OT_4.45_15JUN2018	13,74%	12,25%	12,33%	8,97%	8,74%	7,86%	8,01%
OT_4.75_14JUN2019	9,56%	12,55%	12,76%	11,61%	11,36%	9,78%	10,31%
OT_4.8_15JUN2020	10,51%	9,83%	11,52%	10,47%	11,39%	10,37%	11,07%
OT_3.85_15APR2021	8,59%	8,20%	9,65%	9,88%	9,81%	10,70%	13,64%
OT_2.2_17OCT2022	-	-	-	-	2,76%	3,47%	4,10%
OT_4.95_25OCT2023	8,45%	8,49%	9,61%	9,09%	8,67%	8,19%	8,68%
OT_5.65_15FEB2024	3,75%	9,43%	11,51%	14,07%	13,92%	12,57%	13,02%
OT_2.875_15OCT2025	-	-	-	8,06%	9,89%	9,74%	10,26%
OT_2.875_21JUL2026	-	-	-	-	-	5,15%	6,24%
OT_3.875_15FEB2030	-	-	3,73%	4,40%	4,38%	4,45%	4,41%
OT_4.1_15APR2037	6,63%	6,86%	7,62%	6,98%	8,03%	7,21%	7,69%
OT_4.1_15FEB2045	-	-	-	2,49%	2,48%	2,67%	2,57%

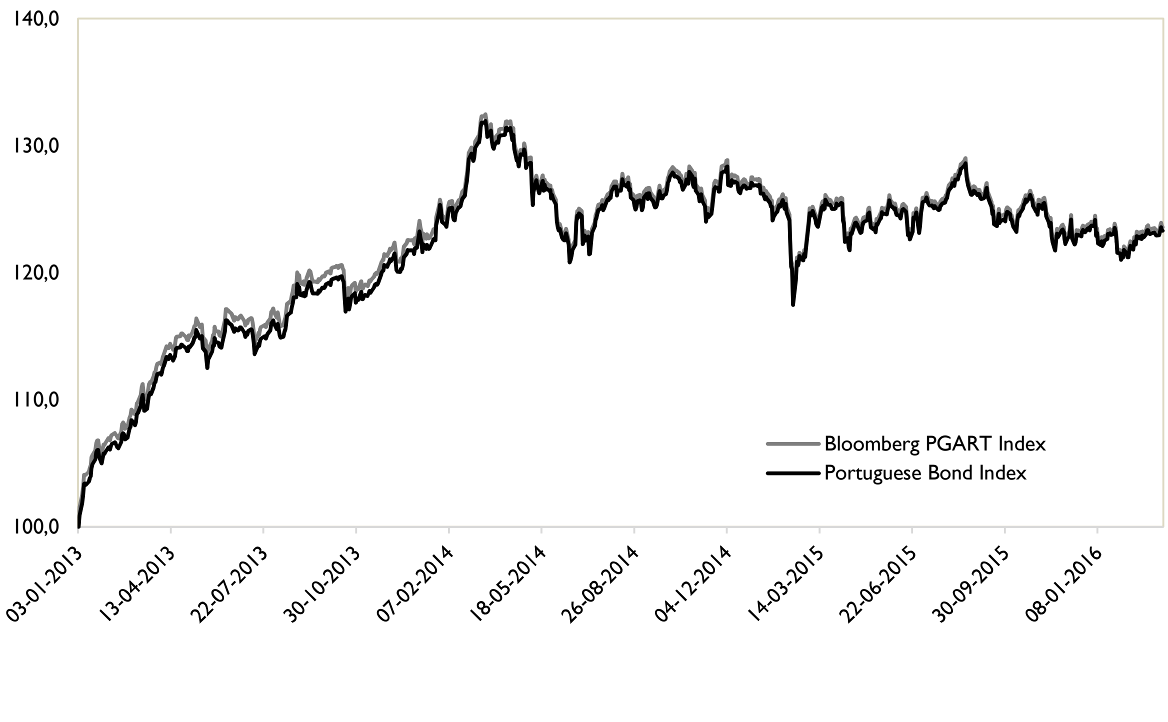
6.6 PBI compared to Bloomberg index PTGATR

In the graph displayed below we present the performance comparison, since 01-01-2013, between the Portuguese Bond index computed in this study and the Blomberg index PTGATR provided by Bloomberg terminal.

We recall that PTGATR includes the Portuguese government bonds with maturity higher than 1 year. This index is computed by Bloomberg using the EFFAS standardized rules for total return indexes[29].

As both indexes have the same methodology (despite some slight differences in implementation) we expected very similar results for both of them. Those results are achieved during the time span analysed as the indexes evolution are almost identical.

Figure 9 – PBI and Bloomberg PGART Index evolution



[29] More information can be obtained in Bloomberg consulting note: Bloomberg/EFFAS Government Bond Indices.

6.7 Total return, clean price and gross price indexes performance

The next figure expresses the evolution of PBI performance, from January 2014 to March 2017, considering the three index calculation methodologies considered in this work: total return index, clean price index and gross price index.

It is clear the importance of including in the index the income received over time from coupons. The clean and gross price indexes only show capital performance, ignoring any proceeds received from coupons, and therefore their performance is substantial lower than the one obtained by the total return methodology.

It is also perceived that gross price index always stand above the clean price index, which is expected as a consequence of accrued interest accountability.

Figure 10 – Total return index vs clean price and dirty price indexes